Solution To Commutative Algebra Sharp

We demonstrate the consistency of the Einstein equations at the level of shock-waves by proving the existence of shock wave solutions of the spherically symmetric Einstein equations for a perfect fluid, starting from initial density and velocity profiles that are only locally of bounded total variation. For these solutions, the components of the gravitational metric tensor are only Lipschitz continuous at shock waves, and so it follows that these solutions satisfy the Einstein equations, as well as the relativistic compressible Euler equations, only in the weak sense of the theory of distributions. The analysis introduces a locally inertial Glimm scheme that exploits the locally flat character of spacetime, and relies on special properties of the relativistic compressible Euler equations when $p=\sigma^2\rho$, $\sigma\equiv const$. We demonstrate the consistency of the Einstein equations at the level of shock-waves by proving the existence of shock wave solutions of the spherically symmetric Einstein equations for a perfect fluid, starting from initial density and velocity profiles that are only locally of bounded total variation. For these solutions, the components of the gravitational metric tensor are only Lipschitz continuous at shock waves, and so it follows that these solutions satisfy the Einstein equations, as well as the relativistic compressible Euler equations, only in the weak sense of the theory of distributions. The analysis introduces a locally inertial Glimm scheme that exploits the locally flat character of spacetime, and relies on special properties of the relativistic compressible Euler equations when $p=\sigma^2\rho$, $\sigma\equiv const$.

This book can be understood as a model for teaching commutative algebra, and takes into account modern developments such as algorithmic and computational aspects. As soon as a new concept is introduced, the authors show how the concept can be worked on using a computer. The computations are exemplified with the computer algebra system Singular, developed by the authors. Singular is a special system for polynomial computation with many features for global as well as for local commutative algebra and algebraic geometry. The book includes a CD containing Singular as well as the examples and procedures explained in the book.

The focus of this book is on algebro-geometric solutions of completely integrable nonlinear partial differential equations in (1+1)-dimensions, also known as soliton equations. Explicitly treated integrable models include the KdV, AKNS, sine-Gordon, and Camassa-Holm hierarchies as well as the classical massive Thirring system. An extensive treatment of the class of algebro-geometric solutions in the stationary as well as time-dependent contexts is provided. The formalism presented includes trace formulas, Dubrovin-type initial value problems, Baker-Akhiezer functions, and theta function representations of all relevant quantities involved. The book uses techniques from the theory of differential equations, spectral analysis, and elements of algebraic geometry (most notably, the theory of compact Riemann surfaces). The presentation is rigorous, detailed, and self-contained, with ample background material provided in various appendices. Detailed notes for each chapter together with an exhaustive bibliography enhance the presentation offered in the main text.

Introductory account of commutative algebra, aimed at students with a background in basic algebra.

Homological Questions in Local Algebra

Manuscripta Mathematica

Proceedings of the Fifth International Fez Conference on Commutative Algebra and Applications, Fez, Morocco, June 23–28, 2008

Introduction To Commutative Algebra

Number Theory

Steps in Commutative Algebra

This unique book on commutative algebra is divided into two parts in order to facilitate its use in several types of courses. The first introductory part covers the basic theory, connections with algebraic geometry, computational aspects, and extensions to module theory. The more advanced second part covers material such as associated primes and primary decomposition, local rings, M-sequences and Cohen-Macaulay modules, and homological methods.

This volume is dedicated to Robert F. Tichy on the occasion of his 60th birthday. Presenting 22 research and survey papers written by leading experts in their respective fields, it focuses on areas that align with Tichy's research interests and which he significantly shaped, including Diophantine problems, asymptotic counting, uniform distribution and discrepancy of sequences (in theory and application), dynamical systems, prime numbers, and actuarial mathematics. Offering valuable insights into recent developments in these areas, the book will be of interest to researchers and graduate students engaged in number theory and its applications.

This book provides a concise yet comprehensive and self-contained introduction to Grobner basis theory and its applications to various current research topics in commutative algebra. It especially aims to help young researchers become acquainted with fundamental tools and techniques related to Grobner bases which are used in commutative algebra and to arouse their interest in exploring further topics such as toric rings, Koszul and Rees algebras, determinantal ideal theory, binomial edge ideals, and their applications to statistics. The book can be used for graduate courses and self-study. More than 100 problems will help the readers to better understand the main theoretical results and will inspire them to further investigate the topics studied in this book.

This book presents an account of several conjectures arising in commutative algebra from the pioneering work of Serre and Auslander-Buchsbaum. The approach is via Hochster's 'Big Cohen-Macaulay modules', though the complementary view point of Peskine-Szpiro and Roberts, who study the homology of certain complexes, is not neglected. Various refinements of Hochster's construction, obtained in collaboration with Bartijn, are included. A special feature is a long chapter written by Van den Dries which explains how a certain type of result can be 'lifted' from prime characteristic to characteristic zero. Though this is primarily a research monograph, it does provide introductions to most of the topics treated. Non-experts may therefore find it an appealing guide into an active area of algebra.

The Geometry of Syzygies

Soliton Equations and their Algebro-Geometric Solutions: Volume 1, (1+1)-Dimensional Continuous Models

Commutative Algebra and Noncommutative Algebraic Geometry

A Term of Commutative Algebra

Geometry of Quantum Theory

Abstract Algebra

The present volume contains a selection of refereed papers from the MEGA-94 symposium held in Santander, Spain, in April 1994. They cover recent developments in the theory and practice of computation in algebraic geometry and present new applications in science and engineering, particularly computer vision and theory of robotics. The volume will be of interest to researchers working in the areas of computer algebra and symbolic computation as well as to mathematicians and computer scientists interested in gaining access to these topics.

This is the second of two volumes of a state-of-the-art survey article collection which emanates from three commutative algebra sessions at the 2009 Fall Southeastern American Mathematical Society Meeting at Florida Atlantic University. The articles reach into diverse areas of commutative algebra and build a bridge between Noetherian and non-Noetherian commutative algebra. The current trends in two of the most active areas of commutative algebra are presented: non-noetherian rings (factorization, ideal theory, integrality), advances from the homological study of noetherian rings (the local theory, graded situation and its interactions with combinatorics and geometry). This second volume discusses closures, decompositions, and factorization.

This book gives an up-to-date exposition on the theory of oblique derivative problems for elliptic equations. The modern analysis of shock reflection was made possible by the theory of oblique derivative problems developed by the author. Such problems also arise in many other physical situations such as the shape of a capillary surface and problems of optimal transportation. The author begins the book with basic results for linear oblique derivative problems and work through the theory for quasilinear and nonlinear problems. The final chapter discusses some of the applications. In addition, notes to each chapter give a history of the topics in that chapter and suggestions for further reading.

During the last few years, several fairly systematic nonlinear theories of generalized solutions of rather arbitrary nonlinear partial differential equations have emerged. The aim of this volume is to offer the reader a sufficiently detailed introduction to two of these recent nonlinear theories which have so far contributed most to the study of generalized solutions of nonlinear partial differential equations, bringing the reader to the level of ongoing research. The essence of the two nonlinear theories presented in this volume is the observation that much of the mathematics concerning existence, uniqueness regularity, etc., of generalized solutions for nonlinear partial differential equations can be reduced to elementary calculus in Euclidean spaces, combined with elementary algebra in quotient rings of families of smooth functions on Euclidean spaces, all of that joined by certain asymptotic interpretations. In this way, one avoids the complexities and difficulties of the customary functional analytic methods which would involve sophisticated topologies on various function spaces. The result is a rather elementary yet powerful and far-reaching method which can, among others, give generalized solutions to linear and nonlinear partial differential equations previously unsolved or even unsolvable within distributions or hyperfunctions. Part 1 of the volume discusses the basic limitations of the linear theory of distributions when dealing with linear or nonlinear partial differential equations, particularly the impossibility and degeneracy results. Part 2 examines the way Colombeau constructs a nonlinear theory of generalized functions and then succeeds in proving quite impressive existence, uniqueness, regularity, etc., results concerning generalized solutions of large classes of linear and nonlinear partial differential equations. Finally, Part 3 is a short presentation of the nonlinear theory of Rosinger, showing its connections with Colombeau's theory, which it contains as a particular case.

Closures, Finiteness and Factorization

Progress in Commutative Algebra 2

Smooth Manifolds and Observables

Introduction To Commutative Algebra, Student Economy Edition

Algebra, Algebraic Topology and their Interactions

A Singular Introduction to Commutative Algebra

This book is designed to be read by students who have had a first elementary course in general algebra. It provides a common generalization of the primes of arithmetic and the points of geometry. The book explains the various elementary operations which can be performed on ideals.

A great many of the objects investigated in mathematics turn out to be groups. These include familiar number systems, such as the integers, the rational numbers, the real numbers, and the complex numbers under addition, as well as the non-zero rationals, reals, and complex numbers, under multiplication. Another important example is given by non-singular matrices under multiplication, and more generally, invertible functions under composition. Group theory allows for the properties of these systems and many others to be investigated in a more general setting, and its results are widely applicable. Group theory is also a rich source of theorems in its own right. Groups underlie many other algebraic structures such as fields and vector spaces. They are also important tools for studying symmetry in all its forms; the principle that the symmetries of any object form a group is foundational for much mathematics. For these reasons, group theory is an important area in modern mathematics, and also one with many applications to mathematical physics. This book presents the latest research in the field.

This introduction to polynomial rings, Gröbner bases and applications bridges the gap in the literature between theory and actual computation. It details numerous applications, covering fields as disparate as algebraic geometry and financial markets. To aid in a full understanding of these applications, more than 40 tutorials illustrate how the theory can be used. The book also includes many exercises, both theoretical and practical.

There is no shortage of books on Commutative Algebra, but the present book is different. Most books are monographs, with extensive coverage. There is one notable exception: Atiyah and Macdonald's 1969 classic. It is a clear, concise, and efficient textbook, aimed at beginners, with a good selection of topics. So it has remained popular. However, its age and flaws do show. So there is need for an updated and improved version, which the present book aims to be.

Solving Systems of Polynomial Equations

Festschrift in Honour of Robert F. Tichy's 60th Birthday

Existence and Consistency by a Locally Inertial Glimm Scheme

Number Theory - Diophantine Problems, Uniform Distribution and Applications

Durham 1981

Integrability and Nonintegrability in Geometry and Mechanics

This is the first of two volumes of a state-of-the-art survey article collection which emanates from three commutative algebra sessions at the 2009 Fall Southeastern American Mathematical Society Meeting at Florida Atlantic University meeting. The articles reach into diverse areas of commutative algebra and build a bridge between Noetherian and non-Noetherian commutative algebra. The current trends in two of the most active areas of commutative algebra are presented: non-noetherian rings (factorization, ideal theory, integrality), advances from the homological study of noetherian rings (the local theory, graded situation and its interactions with combinatorics and geometry). This fist volume concentrates on combinatorics and homology.

The Indian National Science Academy on the occasion ofthe Golden Jubilee Celebration (Fifty years of India's Independence) decided to publish a number of monographs on the selected fields. The editorial board of INS A invited us to prepare a special monograph in Number Theory. In reponse to this assignment, we invited several eminent Number Theorists to contribute expository/research articles for this monograph on Number Theory. Al though some ofthose invited, due to other preoccupations-could not respond positively to our invitation, we did receive fairly encouraging response from many eminent and creative number theorists throughout the world. These articles are presented herewith in a logical order. We are grateful to all those mathematicians who have sent us their articles. We hope that this monograph will have a significant impact on further development in this subject. R. P. Bambah v. C. Dumir R. J. Hans-Gill A Centennial History of the Prime Number Theorem Tom M. Apostol The Prime Number Theorem Among the thousands of discoveries made by mathematicians over the centuries, some stand out as significant landmarks. One of these is the prime number theorem, which describes the asymptotic distribution of prime numbers. It can be stated in various equivalent forms, two of which are: x (I) K(X) '" -I - as x -~+ 00, ogx and Pn '" n log n as n -~+ 00. (2) In (1), K(X) denotes the number of primes P ::s x for any x > O.

This ENCYCLOPAEDIA OF MATHEMATICS aims to be a reference work for all parts of mathe matics. It is a translation with updates and editorial comments of the Soviet Mathematical Encyclopaedia published by 'Soviet Encyclopaedia Publishing House' in five volumes in 1977-1985. The annotated translation consists of ten volumes including a special index volume. There are three kinds of articles in this ENCYCLOPAEDIA. First of all there are survey-type articles dealing with the various main directions in mathematics (where a rather fine subdivi sion has been used). The main requirement for these articles has been that they should give a reasonably complete up-to-date account of the current state of affairs in these areas and that they should be maximally accessible. On the whole, these articles should be understandable to mathematics students in their first specialization years, to graduates from other mathematical areas and, depending on the specific subject, to specialists in other domains of science, en gineers and teachers of mathematics. These articles treat their material at a fairly general level and aim to give an idea of the kind of problems, techniques and concepts involved in the area in question. They also contain background and motivation rather than precise statements of precise theorems with detailed definitions and technical details on how to carry out proofs and constructions. The second kind of article, of medium length, contains more detailed concrete problems, results and techniques.

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Generalized Solutions of Nonlinear Partial Differential Equations

Algorithms in Algebraic Geometry and Applications

Progress in Commutative Algebra 1

Oblique Derivative Problems For Elliptic Equations

Computational Commutative Algebra 1

Shock-Wave Solutions of the Einstein Equations with Perfect Fluid Sources: Existence and Consistency by a Locally Inertial Glimm Scheme

Approach your problems from the right end It isn't that they can't see the solution. It is and begin with the answers. Jhen one day, that they can't see the problem. perhaps you will find the final question. G. K. Chesterton. The Scandal of Father 'The Hermit Oad in Crane Feathers' in R. Brown 'The point of a Pin' . • 1111 Oulik'. n. . Chi" •. • ~ Mm~ Mu.d. ", Growing specialization and diversification have brought a host of monographs and textbooks on increasingly specialized topics. However, the "tree" of knowledge of mathematics and related fields does not grow only by putting forth new branches. It also happens, quite often in fact, that branches which were thought to be completely disparate are suddenly seen to be related. Further, the kind and level of sophistication of mathematics applied in various sciences has changed drastically in recent years: measure theory is used (non-trivially) in regional and theoretical economics; algebraic geometry interacts with physics; the Minkowsky lemma, coding theory and the structure of water meet one another in packing and covering theory; quantum fields, crystal defects and mathematical programming profit from homotopy theory; Lie algebras are relevant to filtering; and prediction and electrical engineering can use Stein spaces. And in addition to this there are such new emerging subdisciplines as "experimental mathematics", "CFD", "completely integrable systems", "chaos, synergetics and large-scale order", which are almost impossible to fit into the existing classification schemes. They draw upon widely different sections of mathematics.

This second edition of a successful graduate text provides a careful and detailed algebraic introduction to Grothendieck's local cohomology theory, including in multi-graded situations, and provides many illustrations of the theory in commutative algebra and in the geometry of quasi-affine and quasi-projective varieties. Topics covered include Serre's Affineness Criterion, the Lichtenbaum-Hartshorne Vanishing Theorem, Grothendieck's Finiteness Theorem and Faltings' Annihilator Theorem, local duality and canonical modules, the Fulton-Hansen Connectedness Theorem for projective varieties, and connections between local cohomology and both reductions of ideals and sheaf cohomology. The book is designed for graduate students who have some experience of basic commutative algebra and homological algebra and also experts in commutative algebra and algebraic geometry. Over 300 exercises are interspersed among the text; these range in difficulty from routine to challenging, and hints are provided for some of the more difficult ones.

This textbook is an introduction to the concept of factorization and its application to problems in algebra and number theory. With the minimum of prerequisites, the reader is introduced to the notion of rings, fields, prime elements and unique factorization. The author shows how concepts can be applied to a variety of examples such as factorizing polynomials, finding determinants of matrices and Fermat's 'two-squares theorem'. Based on an undergraduate course given at the University of Sheffield, Dr Sharpe has included numerous examples which demonstrate how frequently these ideas are useful in concrete, rather than abstract, settings. The book also contains many exercises of varying degrees of difficulty together with hints and solutions. Second and third year undergraduates will find this a readable and enjoyable account of a subject lying at the heart of much of mathematics.

Rapid, concise, self-contained introduction assumes only familiarity with elementary algebra. Subjects include algebraic varieties; products, projections, and correspondences; normal varieties; differential forms; theory of simple points; algebraic groups; more. 1958 edition.

Volume 6: Subject Index — Author Index

Commutative Algebra

Third Edition

Quasianalytic Monogenic Solutions of a Cohomological Equation

Encyclopaedia of Mathematics

Combinatorics and Homology

This book is concerned with the research conducted in the late 1970s and early 1980s in the theory of commutative Neotherian rings. It consists of articles by invited speakers at the Symposium of Commutative Algebra held at the University of Durham in July 1981; these articles are all based on lectures delivered at the Symposium. The purpose of this book is to provide a record of at least some aspects of the Symposium, which several of the world leaders in the field attended. Several articles are included which provide surveys, incorporating historical perspective, details of progress made and indications of possible future lines of investigation. The book will be of interest to scholars of commutative and local algebra.

This ACM volume deals with tackling problems that can be represented by data structures which are essentially matrices with polynomial entries, mediated by the disciplines of commutative algebra and algebraic geometry. The discoveries stem from an interdisciplinary branch of research which has been growing steadily over the past decade. The author covers a wide range, from showing how to obtain deep heuristics in a computation of a ring, a module or a morphism, to developing means of solving nonlinear systems of equations - highlighting the use of advanced techniques to bring down the cost of computation. Although intended for advanced students and researchers with interests both in algebra and computation, many parts may be read by anyone with a basic abstract algebra course.

First textbook-level account of basic examples and techniques in this area. Suitable for self-study by a reader who knows a little commutative algebra and algebraic geometry already. David Eisenbud is a well-known mathematician and current president of the American Mathematical Society, as well as a successful Springer author.

A classic problem in mathematics is solving systems of polynomial equations in several unknowns. Today, polynomial models are ubiquitous and widely used across the sciences. They arise in robotics, coding theory, optimization, mathematical biology, computer vision, game theory, statistics, and numerous other areas. This book furnishes a bridge across mathematical disciplines and exposes many facets of systems of polynomial equations. It covers a wide spectrum of mathematical techniques and algorithms, both symbolic and numerical.The set of solutions to a system of polynomial equations is an algebraic variety - the basic object of algebraic geometry. The algorithmic study of algebraic varieties is the central theme of computational algebraic geometry. Exciting recent developments in computer software for geometric calculations have revolutionized the field. Formerly inaccessible problems are now tractable, providing fertile ground for experimentation and conjecture. The first half of the book gives a snapshot of the state of the art of the topic. Familiar themes are covered in the first five chapters, including polynomials in one variable, Grobner bases of zero-dimensional ideals, Newton polytopes and Bernstein's Theorem, multidimensional resultants, and primary decomposition.The second half of the book explores polynomial equations from a variety of novel and unexpected angles. It introduces interdisciplinary connections, discusses highlights of current research, and outlines possible future algorithms. Topics include computation of Nash equilibria in game theory, semidefinite programming and the real Nullstellensatz, the algebraic geometry of statistical models, the piecewise-linear geometry of valuations and amoebas, and the Ehrenpreis-Palamodov theorem on linear partial differential equations with constant coefficients.Throughout the text, there are many hands-on examples and exercises, including short but complete sessions in MapleR, MATLABR, Macaulay 2, Singular, PHCpack, CoCoA, and SOSTools software. These examples will be particularly useful for readers with no background in algebraic geometry or commutative algebra. Within minutes, readers can learn how to type in polynomial equations and actually see some meaningful results on their computer screens. Prerequisites include basic abstract and computational algebra. The book is designed as a text for a graduate course in computational algebra.

A Second Course in Algebraic Geometry and Commutative Algebra

Proceedings of a Conference held in Stockholm, Aug. 3 - 13, 1983, and later developments

Real Solutions to Equations from Geometry

Sucessful Candidates for the Degrees of D.Phil., B.Litt., and B.Sc. with Titles of Their Theses

Local Cohomology

Commutative Algebra and its Applications

Understanding, finding, or even deciding on the existence of real solutions to a system of equations is a difficult problem with many applications outside of mathematics. While it is hopeless to expect much in general, we know a surprising amount about these questions for systems which possess additional structure often coming from geometry. This book focuses on equations from toric varieties and Grassmannians. Not only is much known about these, but such equations are common in applications. There are three main themes: upper bounds on the number of real solutions, lower bounds on the number of real solutions, and geometric problems that can have all solutions be real. The book begins with an overview, giving background on real solutions to univariate polynomials and the geometry of sparse polynomial systems. The first half of the book concludes with fewnomial upper bounds and with lower bounds to sparse polynomial systems. The second half of the book begins by sampling some geometric problems for which all solutions can be real, before devoting the last five chapters to the Shapiro Conjecture, in which the relevant polynomial systems have only real solutions.

Highly regarded by instructors in past editions for its sequencing of topics as well as its concrete approach, slightly slower beginning pace, and extensive set of exercises, the latest edition of Abstract Algebra extends the thrust of the widely used earlier editions as it introduces modern abstract concepts only after a careful study of important examples. Beachy and Blairs clear narrative presentation responds to the needs of inexperienced students who stumble over proof writing, who understand definitions and theorems but cannot do the problems, and who want more examples that tie into their previous experience. The authors introduce chapters by indicating why the material is important and, at the same time, relating the new material to things from the students background and linking the subject matter of the chapter to the broader picture. Instructors will find the latest edition pitched at a suitable level of difficulty and will appreciate its gradual increase in the level of sophistication as the student progresses through the book. Rather than inserting superficial applications at the expense of important mathematical concepts, the Beachy and Blair solid, well-organized treatment motivates the subject with concrete problems from areas that students have previously encountered, namely, the integers and polynomials over the real numbers. Supplementary material for instructors and students available on the books Web site: www.math.niu.edu/~beachy/abstract_algebra/

We prove that the solutions of a cohomological equation of complex dimension one and in the analytic category have a monogenic dependence on the parameter, and we investigate the question of their quasi analyticity. This cohomological equation is the standard linearized conjugacy equation for germs of holomorphic maps in a neighborhood of a fixed point. The parameter is the eigenvalue of the linear part, denoted by q. Borel's theory of non-analytic monogenic functions has been first investigated by Arnold and Herman in the related context of the problem of linearization of analytic diffeomorphisms of the circle close to a rotation.Herman raised the question whether the solutions of the cohomological equation had a quasi analytic dependence on the parameter q. Indeed they are analytic for $q\in\mathbb{C}\setminus\mathbb{S}^1$, the unit circle $\S^1$ appears as a natural boundary (because of resonances, i.e. roots of unity), but the solutions are still defined at points of $\mathbb{S}^1$ which lie 'far enough from resonances'. We adapt to our case Herman's construction of an increasing sequence of compacts which avoid resonances and prove that the solutions of our equation belong to the associated space of monogenic functions; some general properties of these monogenic functions and particular properties of the solutions are then studied.For instance the solutions are defined and admit asymptotic expansions at the points of $\mathbb{S}^1$ which satisfy some arithmetical condition, and the classical Carleman Theorem allows us to answer negatively to the question of quasi analyticity at these points. But resonances (roots of unity) also lead to asymptotic expansions, for which quasi analyticity is obtained as a particular case of Ecalle's theory of resurgent functions.And at constant-type points, where no quasi analytic Carleman class contains the solutions, one can still recover the solutions from their asymptotic expansions and obtain a special kind of quasi analyticity. Our results are obtained by reducing the problem, by means of Hadamard's product, to the study of a fundamental solution (which turns out to be the so-called q-logarithm or 'quantum logarithm'). We deduce as a corollary of our work the proof of a conjecture of Gammel on the monogenic and quasi analytic properties of a certain number-theoretical Borel-Wolff-Denjoy series.

This book gives an introduction to fiber spaces and differential operators on smooth manifolds. Over the last 20 years, the authors developed an algebraic approach to the subject and they explain in this book why differential calculus on manifolds can be considered as an aspect of commutative algebra. This new approach is based on the fundamental notion of observable which is used by physicists and will further the understanding of the mathematics underlying quantum field theory.

An Algebraic Introduction with Geometric Applications

Second Edition

Gr?bner Bases in Commutative Algebra

Research in British Universities, Polytechnics and Colleges

Introduction to Algebraic Geometry

Focus on Group Theory Research

Available for the first time in soft cover, this book is a classic on the foundations of quantum theory. It examines the subject from a point of view that goes back to Heisenberg and Dirac and whose definitive mathematical formulation is due to von Neumann. This view leads most naturally to the fundamental questions that are at the basis of all attempts to understand the world of atomic and subatomic particles.

The first comprehensive and up-to-date account of discriminant equations and their applications. For graduate students and researchers.

Discriminant Equations in Diophantine Number Theory

Rings and Factorization

Computational Methods in Commutative Algebra and Algebraic Geometry